PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Mode-Locked Semiconductor Diode Lasers [and Discussion]

H. A. Haus and T. S. Moss

Phil. Trans. R. Soc. Lond. A 1980 **298**, 257-266
doi: 10.1098/rsta.1980.0249

Phil. Trans. R. Soc. Lond. A **298**, 257–266 (1980)
Printed in Great Britain

Mode-locked semiconductor diode lasers

By H. A. Haus
Department of Electrical Engineering and Computer Science and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, *U.S.A.*

The reasoning that led to the first successful mode-locking of continuous wave (c.w.) semiconductor laser diodes is described. The theory of forced mode-locking, as modified for semiconductor laser diodes in an external resonator, is outlined. Experiments on GaAlAs and InGaAsP diodes are summarized. The shortest pulses to date are 16 ps f.w.h.m. Limitations on pulse width are discussed and ways of producing shorter pulses are suggested.

1. Introduction

The large absolute bandwidths of optical media are exploited in the generation of picosecond pulses from lasers by the technique of 'mode-locking'. The pulses are shortened each time they pass through the modulation element (active or passive). Even though the shortening of the pulse in one transit may be small, the repetitive action of the modulation element can lead to pulses whose bandwidth can approach that of the laser medium.

The usefulness of picosecond optical pulses for time-resolved spectroscopy needs no elaboration. The use of picosecond optics for optical communications applications, such as signal processing, is currently prevented by the physical size of mode-locked systems. Laser diodes are physically small and are the ideal laser system for optical integration. Previous attempts at 100% modulation of a diode laser at frequencies greater than 1 GHz were limited by the carrier life time. The 'resonance phenomenon' utilized in mode-locking can overcome these limitations. Laser diodes have been mode-locked in the past year (Ho *et al.* 1978 *b*, *c*; Ho 1978) producing pulses as short as 16 ps (Glasser 1979).

The present paper reviews the reasoning that led to the first successful forced mode-locking of a laser diode, gives a brief summary of the theory of forced mode-locking as pertinent to the laser diode, and presents experimental results, in particular with regard to the self pulsing that has been observed so far in all laser diodes in external resonators.

2. Historical background

I developed a theory of passive mode-locking (Haus 1975 *a*, *b*) in the early 1970s and subsequently spent a year at the Bell Laboratories to compare the theory with experiments by Shank and Ippen (Shank & Ippen 1974) then in progress. One of the puzzles that had to be resolved at the time was the production of picosecond pulses in dye laser systems in which the laser and absorber had relaxation times of the order of nanoseconds. The recognition that the pulse was shaped by the joint action of the gain and absorber media (Haus 1975 *b*; New 1974), helped to explain the formation of the pulses, and the analytic theory predicted their shape (Haus *et al.* 1975).

The fact that picosecond pulses can be generated with elements that have nanosecond relaxation times led one to expect that laser diodes may be made to produce short pulses in spite of their inherently long relaxation times. In passively mode-locked dye lasers, both the gain-medium and the loss-medium are dyes of comparable relaxation times (if not bandwidths; the bandwidth of the absorber dye is, generally, narrower). Analogously, one may imagine the use of a diode laser biased above threshold as the gain medium, a diode laser below threshold as the saturable absorber.

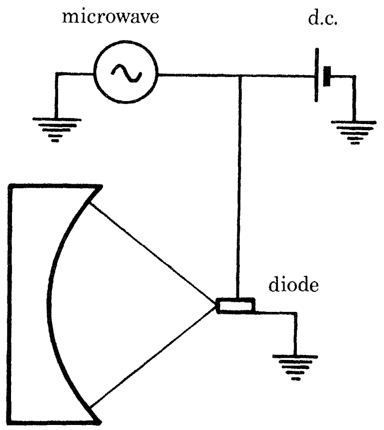

FIGURE 1. Schematic of diode in external resonator.

All mode-locked dye laser systems have resonator round-trip times that are comparable to, or longer than, the relaxation time of the laser medium. The population inversion in the medium is given a chance to recover so that each pulse experiences the full gain (as determined by the pump). In this way the energy in the pulse is maximized, and if the system is passively mode-locked, the saturable absorber is modulated to the utmost. By analogy with mode-locked dye laser operation, the two diodes would have to operate at the two ends of an external resonator formed of mirrors and/or lenses.

With this broad objective, P.-T. Ho started experimenting with one diode laser in an external resonator to ascertain the coupling effectiveness of the external resonator to the diode. After some initial attempts with a lens system that was not anti-reflexion coated and proved excessively lossy, a design was chosen (see figure 1) based on a system operated by A. Mooradian and M. W. Fleming of the Lincoln Laboratory, designed for frequency selectivity. The coupling of the diode to the external resonator was judged from the reduction of the the threshold current and from the appearance of a distinct microwave spectrum at the frequency near $c/2l$ (where l is the length of the external resonator) in the output of the fast detector which monitored the laser power. The spectrum was taken as an indication of beating of the axial modes of the external resonator excited by spontaneous emission noise.

When successful operation of a single diode in an external resonator was demonstrated, it appeared expedient to modulate the bias current at or near the frequency $c/2l$ and force mode-locking of the diode. Because of its size and nature of pumping, the laser diode is unique among lasers in the ease with which the gain can be modulated. The experiment was successful beyond expectation, producing 23 ps pulses, and all work to date has been concentrated on an understanding of the limits imposed on pulse duration and pulse coherence and on ways to overcome them.

3. Theory of forced mode-locking of laser diodes in external resonator

Mode-locking may be explained by using a time domain (Kuizenga & Siegman 1970), or frequency domain, approach. We review briefly the frequency domain analysis of mode-locking (Haus 1975*c*). Figure 2 shows the frequencies of the different axial modes of the resonator. The spacing of the modes is $\Delta\omega = 2\pi/T_R$, where T_R is the round-trip time in the resonator. The height of the vertical lines shows their loss. The laser medium introduces a frequency dependent gain with a profile as shown. If the electric field of the nth mode has a (normalized) excitation ampli-

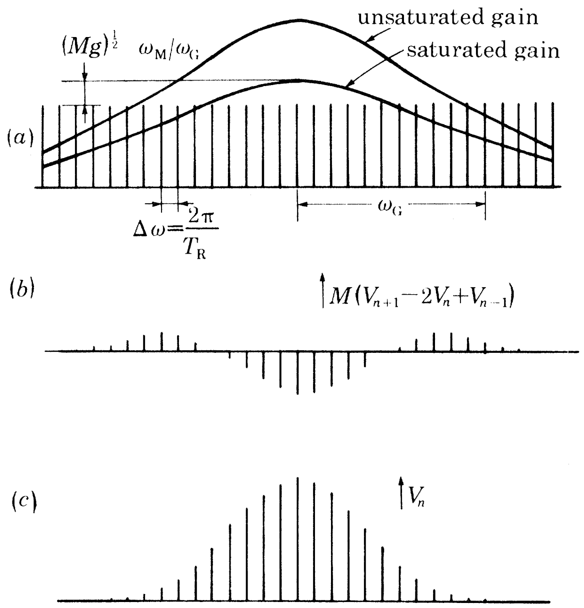

Figure 2. The mode spectrum of single Fabry–Perot resonator: (*a*) the loss and gain as functions of ω; (*b*) the injection signal; (*c*) the spectrum of mode-locked pulse.

tude V_n at frequency ω_n, then a sinusoidally time-dependent loss (normalized to unity minimum value) $1 + 2M(1 - \cos \omega_M t)$ in the resonator produces injection signals at $\omega_n \pm \omega_M$. When ω_M is chosen to correspond to the mode spacing $2\pi/T_R$, the injection signals drive the adjacent modes. The gain of each mode is not equal to the loss of the mode because the injection signals can make up the difference. A self-consistent solution of the equations evaluates the amplitudes V_n in the presence of these injection signals. These amplitudes are also shown schematically in figure 2 and are proportional to (Haus 1975*c*) $M(V_{n+1} - 2V_n + V_{n-1})$. They can be approximated by the second derivative of $V(\omega)$ treated as a continuous function. The gain of the central modes is higher than their loss; the injection signals are in antiphase for these modes. The gain of the modes in the wings is less than their loss; the injection signals are in phase with the mode amplitudes to make up for this difference. A self-consistent solution of the equation gives a *required* (normalized) excess gain g at line centre that is given by (Haus 1975*c*)

$$g - 1 = (Mg)^{\frac{1}{2}}\omega_M/\omega_G, \tag{3.1}$$

where ω_G is the linewidth of the gain, and M is the modulation coefficient of the forced loss modulation. The required excess gain is the higher the stronger the modulation and the narrower the laser linewidth. In the absence of noise, the optical power reaches a value such that the gain decreases to the required level.

The theory of forced mode-locking has to be reinterpreted when applied to the diode laser in an external resonator. Four new factors have to be taken into account: (*a*) that the forced modulation is one of gain, not loss; (*b*) that the pulse is intense enough to modulate the gain in its own right; (*c*) that the spontaneous emission is strong; (*d*) the effect of the composite Fabry–Perot resonator.

(*a*) *Forced gain modulation*

In the theory of forced mode-locking by loss modulation, the time-dependent normalized loss, $1+2M(1-\cos\omega_{\mathrm{M}}t)$, multiplying the mode, $V_n \exp j\omega_n t$, produces two sidebands. The bandwidth of the gain limits the spectrum of the pulse. When the gain is modulated, the sideband injection and bandwidth limiting occur in one and the same element and hence, strictly speaking, the existing theory of forced mode-locking does not apply. However, in general, the modulation is not very strong and the linewidth of the laser is large. One may expand the equation to first order in the modulation coefficient and the gain-profile frequency dependence $(\omega_n-\omega_0)^2/\omega_{\mathrm{L}}^2$. In this limit the effects are additive and hence the previously developed equations for loss modulation may be applied. Of course the peak of the pulse now occurs at the instant of the gain maximum.

(*b*) *Modulation of gain medium*

The intensities in a diode laser at a few milliwatts average power are such that the gain medium is saturated appreciably. A time-dependent laser output produces time dependence of the gain. The theory of forced mode-locking postulates a modulation (M) of the gain, independent of the optical power. In general, saturation of the gain tends to counteract the 'forced' gain modulation, because the 'wings' of the pulse experience more gain than the centre of the pulse. In the mode-locked diode laser system, the round-trip time, T_{R}, is short compared with the laser medium relaxation time, T_{L}, and the pulse duration, τ_{p}, is very much shorter. The population inversion cannot follow the pulse envelope. The most strongly excited Fourier component of the population inversion is the fundamental at $2\pi/T_{\mathrm{R}}$: the harmonics of order n are reduced by approximately a factor $1/n$. The fundamental-frequency component of the gain modulation may be separated into a part that is in phase with the peak of the pulse and a quadrature part. The in-phase part counteracts the forced modulation, while the quadrature part is mainly responsible for a time advance of the pulse. Hence only the in phase component needs to be included in the effective loss-modulation coefficient.

A design criterion may be 'gleaned' from the above. Consider the equation for the carrier density, n (Kressel & Butler 1977):

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -\frac{n}{\tau_n}+\frac{J}{qd}-\alpha n p, \tag{3.2}$$

where τ_n is the relaxation time of the carriers, J is the current density in the junction, d is its width, α is a proportionality constant and p is the photon density. The gain is approximately proportional to n. If one assumes that the gain modulation is small, the pulse is short (pulse width $\tau_{\mathrm{p}} \ll T_{\mathrm{R}}$), then one finds for the fundamental component of the perturbation $\tilde{n}_1$ ($n = n_0+\tilde{n}_1$) at frequency $\omega_{\mathrm{M}} \approx 2\pi/T_{\mathrm{R}}$,

$$\tilde{n}_1 = \frac{\tilde{J}_1/qd-\alpha n_0\tilde{p}_1}{j\omega_{\mathrm{M}}+\tau_{\mathrm{eff}}^{-1}}, \tag{3.3}$$

where

$$\tau_{\mathrm{eff}}^{-1} = \tau_n^{-1}+\alpha p_0. \tag{3.4}$$

The fundamental component of p, $\tilde{p}_1$, is equal to the time average component, p_0, if τ_p is short compared with T_R.

Now, the carrier modulation due to $\tilde{J}_1$ is, according to (3.2),

$$\left|\frac{\tilde{n}_1}{n_1}\right| = \left|\frac{\tilde{J}_1}{qd}\right| (\omega_M^2 + \tau_{eff}^{-2})^{-\frac{1}{2}}. \tag{3.5}$$

From this modulation, we have to subtract the in-phase component of the modulation due to $\tilde{p}_1$. Now

$$\alpha n_0 \, |\tilde{p}_1| \approx \alpha n_0 \, p_0 = \frac{J_0}{qd} - \frac{1}{\tau_n} = \frac{J_0 - J_{th}}{qd}, \tag{3.6}$$

where J_{th} is the threshold current density.

Pulse forming is possible only if (3.5) is greater than the in-phase component of the carrier modulation by $\tilde{p}_1$:

$$\left|\frac{\tilde{J}_1}{J_0}\right| > \frac{1 - J_{th}/J_0}{(\omega_M^2 \tau_{eff}^2 + 1)^{\frac{1}{2}}}. \tag{3.7}$$

This shows that, for a given current modulation, mode-locking (i) requires that the diode not be operated too far above threshold, (ii) is facilitated if $\omega_M \tau_{eff}$ is chosen larger than unity: for the same current modulation, diodes in short resonators are more easily mode-locked. It should be emphasized, however, that the above argument assumed that gain saturation is only a correction to the gain modulation by the driving current.

(*c*) *Spontaneous emission noise*

The spontaneous emission noise is of greater importance in laser diodes than in other lasers as evidenced by the relatively round knee of the power–excitation (J) curve; the onset of threshold is not abrupt. Spontaneous emission noise tends to counteract mode-locking (Haus & Ho 1979) just as noise interferes with the locking of simple oscillators (Kurokawa 1973). Noise is also a basic ingredient in the locking of modes in clusters in the composite Fabry–Perot resonator of a laser diode in an external resonator, as explained in the following.

(*d*) *The composite Fabry–Perot resonator effect*

Different gains are experienced by the axial modes of a composite Fabry–Perot resonator, such as the system of figure 3, with no coating on the internal end face of the diode. A simple way of analysing the system is to compute the reflexion coefficient, Γ, of a wave impinging upon the internal diode-endface. The gain is proportional to $|\Gamma| - 1$. The reflexion coefficient is a periodic function of frequency ω, with period $\omega_D = \pi c/N l_d$, where N is the refractive index of the diode material and l_d is the diode's length. If the gain maxima are sufficiently high – the frequency dependent loss sufficiently large – then the net gain peaks are separated by 'deep valleys' of loss. An axial mode cluster near any one of the gain maxima is mode-locked independently of the other clusters. In the absence of noise, the net power in the laser reaches a level such that the (maximum) gain at line centre is equal to the gain required to lock the cluster at line centre. Only this one mode cluster would oscillate. If the system is driven by noise, the gain is less than that required for steady-state oscillation. The excitation of each cluster is proportional to the noise drive and inversely proportional to the difference between the *required* gain and *actual* gain levels.

The clusters are excited by statistically independent noise sources in different portions of the spontaneous emission spectrum; the gain responds only to the net fluctuations of the power. The *net* fluctuations are greatly reduced by incoherent superposition so that the spectra of the clusters are (approximately) statistically independent. The slight slope of the actual gain profile causes a detuning of the carrier frequency of the mode-locked cluster from the frequency of the loss minimum.

An analysis of the reflexion coefficient, Γ, of a wave incident upon the diode shows that the bandwidth of $|\Gamma|-1$ is proportional to $\pi c/nl_d$. The bandwidth narrows with increasing peak value of $|\Gamma|-1$. Because the pulse width of a mode-locked cluster is proportional to the inverse square root of the bandwidth, reduction of $|\Gamma|-1$ is at a premium. The loss of the pulse in one round trip in the external resonator must be kept as small as possible.

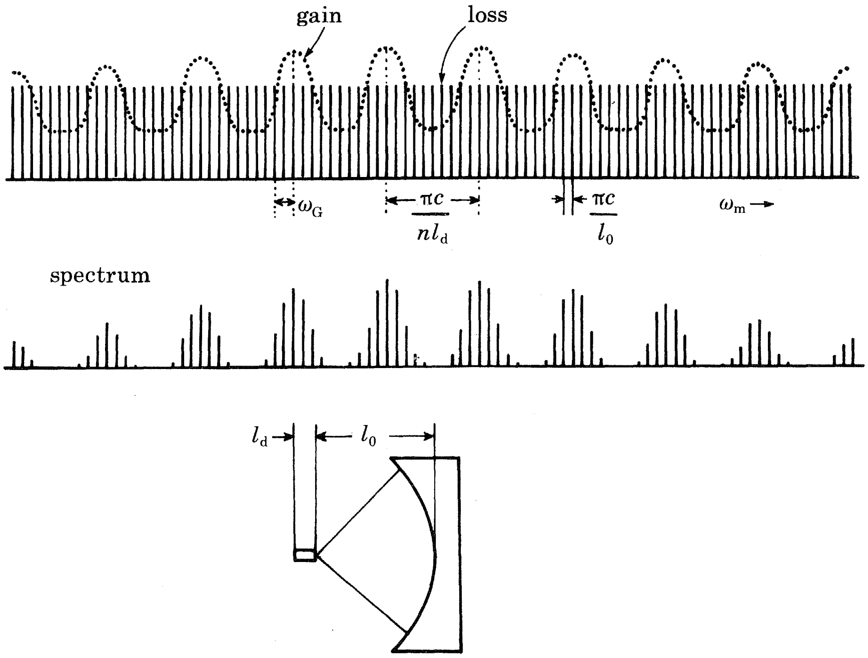

FIGURE 3. The mode spectrum of composite Fabry–Perot resonator; loss and gain as functions of ω.

An incoherent superposition of noise pulses, when measured by second harmonic generation (s.h.g.), gives a broad background, a pedestal whose width is proportional to the width of the envelope and a 'coherence' spike in the centre (Ippen & Shank 1977). If the two pulse replicas superimposed in the second harmonic crystal are of the same intensity, the ratios are 3:2:1 for the respective levels of coherence spike, peak of pedestal, and background. If the noise spectra have carriers that are displaced by $\omega_D = \pi c/Nl_d$, the central coherence spike is repeated after delays that are multiples of $2nl_d/c$. In the next section we present experiments that are in general agreement with the above predictions, but exhibit also some unexplained features.

4. The experimental results

A schematic of the experimental set-up is shown in figure 4. The second harmonic is generated in a $LiIO_3$ crystal. Most components are conventional. Note the use of a second harmonic blocking filter, which prevents the second harmonic generated in the laser diode from entering the photomultiplier (p.m.t.). Part of the beam is detected in a fast photodiode and fed to a microwave spectrum analyser. Another part is detected directly on a fast oscilloscope that displays the pulse train without resolving the individual pulses.

A second harmonic trace for a InGaAsP diode operating at 1.2 μm is shown in figure 5. The intensities in the two arms of the interferometer A and B were adjusted to be equal. The width of the pulse envelope is inferred to be 18 ps under the assumption of a Gaussian pulse. Note the characteristic structure of background, pedestal and repeated coherence spike as predicted in

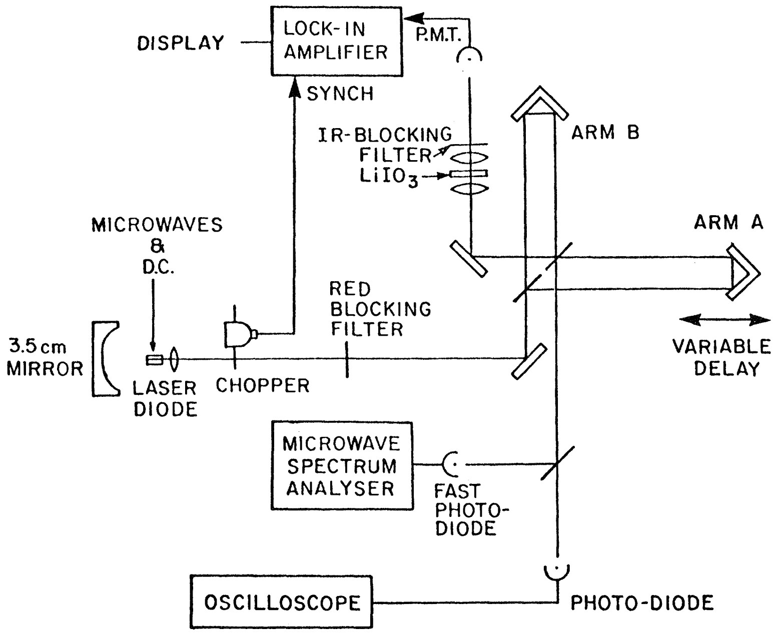

FIGURE 4. Schematic of measurement set-up.

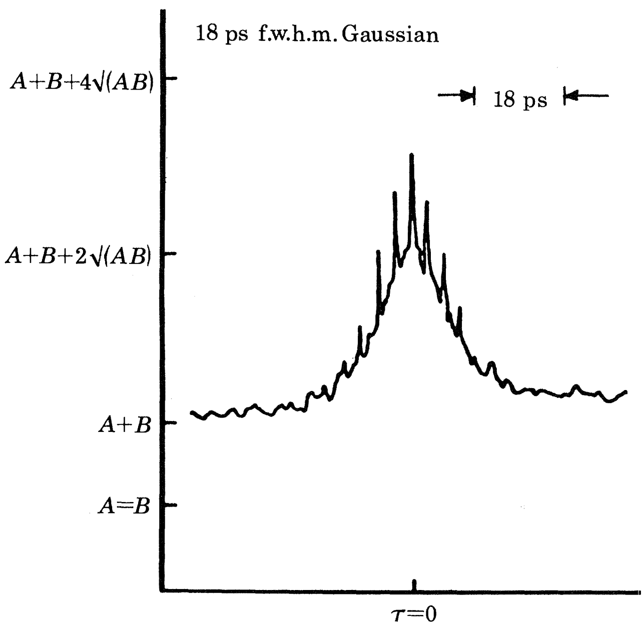

FIGURE 5. An s.h.g. trace of mode-locked InGaAsP diode operating at 1.2 μm.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

the previous section. Their ratio is roughly 3:2:1. The height of the spikes is somewhat lower because of the limited temporal resolution of the recorder.

Table 1 shows a summary of diode types mode-locked so far (Ho *et al.* 1978*b*; Glasser 1978, 1979) and the pulse widths achieved. The shortest pulse width was 16 ps. This particular width was attained with a diode that had its internal face anti-reflexion coated. The periodic superstructure was reduced by the anti-reflexion coating, but not eliminated. The persistence of the superstructure is an indication that it is difficult to achieve low reflexion in the external resonator geometry of figure 1 in which the extreme beam divergence introduces a large spread of angles of incidence upon the diode face, not all of which can be matched.

Table 1. Diode types and widths of mode-locked pulses
(Measured from s.h.g. pedestal, assuming Gaussian envelope.)

material	wavelength/nm	repetition rate/GHz	pulse length/ps
GaAlAs	810	1.5 and 3.0	23
GaInAsP	1310	1.5 and 2.1	28
GaInAsP	1210	1.5 and 2.1	16

Average optical output power: *ca.* 1 mW.

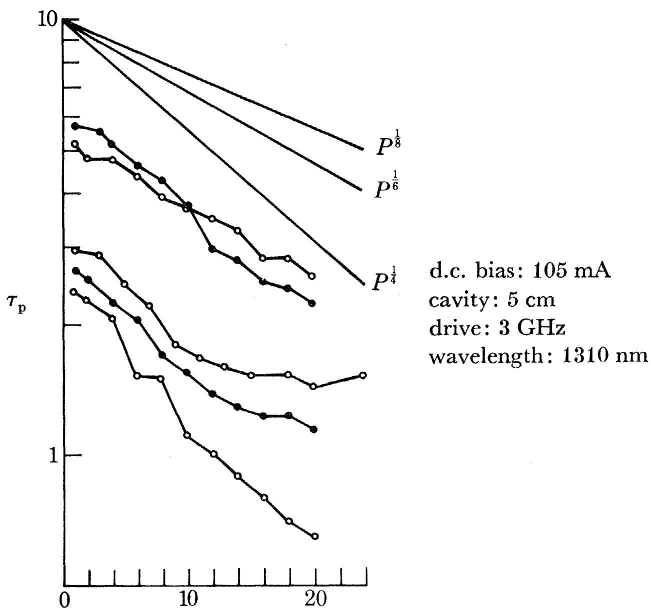

Figure 6. The inverse pulse width (as measured by second harmonic signal divided by the square of the d.c. detector current, i.e. the square of the average optical power) plotted against microwave modulation power, P.

One may estimate the pulse width (f.w.h.m.) from the modulation, M, of the gain coefficient width of the gain–frequency curve. The latter is proportional to the diode round-trip time that shows up directly in the s.h.g. trace. With a modulation frequency of 4 GHz one predicts a f.w.h.m. of *ca.* 23 ps/$M^{\frac{1}{4}}$ (Haus 1979). The dependence on M is very weak and hence $M^{\frac{1}{4}}$ is of the order of unity. It is interesting that the correct order of magnitude for the pulse width is predicted by this analysis. This suggests that major reductions in pulse width are not to be expected in the existing system. Yet not all experimental data are predicted by the above theory.

Figure 6 shows the inverse pulse width τ_p^{-1} as a function of modulation coefficient, M, which is proportional to the square root of the microwave power, P. In all experiments, τ_p^{-1} varied more rapidly than $P^{\frac{1}{8}}$, the dependence predicted by conventional mode-locking theory. The experimental data tend to lie closer to a $P^{\frac{1}{4}}$ dependence.

Finite pulse widths are observed even for very small microwave powers. In fact pulses are observed even when the microwave power is completely turned off; the laser diode 'self pulses'. Figure 7 shows an s.h.g. obtained on the same diode with zero modulation drive. A pulse envelope width of 27 ps is inferred. The periodic superstructure is less pronounced.

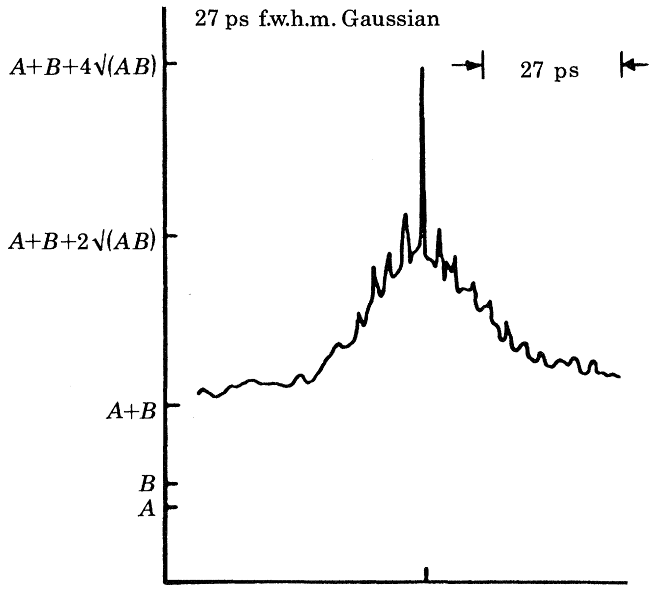

FIGURE 7. A s.h.g. trace of the same diode as in figure 5 with no mode-locking drive.

The phenomenon of self pulsing is, as yet, incompletely understood. Self pulsing has been observed on laser diodes early in their development (Paoli 1975). The diodes used in our experiments did not self pulse if the external resonator was blocked. A stability analysis of the diode in an external resonator, with the use of the conventional diode equation (3.2), shows no instability, but does yield new characteristic (complex) frequencies (Glasser 1979); the decay constants of these frequencies are smaller than those of the free-running diode with the same average power. This is an indication that the external resonator may encourage a tendency towards an instability if there are effects that destabilize the diode operation. Among these the following seem possible candidates: (*a*) saturable loss in the unexcited end-regions of the stripe; (*b*) saturable traps (Copeland 1978); (*c*) the mode beating proposed by Paoli and Ripper.

5. Conclusions

I have presented a brief review of mode-locking theory, taking into account the special properties of the laser diode in an external resonator. Several tentative conclusions were reached.

The previously developed theories of mode-locking by loss modulation are applicable to mode-locking by gain modulation if the depth of modulation is small, as is normally true.

Further, gain modulation by a given current drive is counteracted by gain saturation. The counteraction is least when the diode is operated close to threshold.

The composite Fabry–Perot effect produces mode-locking in clusters that account for the superstructure in the s.h.g. trace. The bandwidth limitation produced by this effect leads to a predicted pulse width of the order of 23 ps, in general agreement with observation. Experimentally, one observes two effects that need further clarification: the pulse width does not follow the $M^{-\frac{1}{4}}$ law, and the diode self pulses when placed in the external resonator.

Further progress in the mode-locking of semiconductor diodes will be achieved only when the composite Fabry–Perot effect is overcome. The best chance for this is offered by an integrated optics version of the diode–external resonator configuration in which the reflexion of the diode–resonator interface is suppressed.

The experiments reported here were performed by Dr P.-T. Ho and Dr L. A. Glasser as part of their thesis work. The second harmonic generation was made possible through the expertise of Dr E. P. Ippen from whose collaboration we benefited immeasurably during his stay at M.I.T. as a visiting professor.

References (Haus)

Copeland, J. A. 1978 *Electron. Lett.* **14**, 809–810.
Glasser, L. A. 1978 *Electron. Lett.* **14**, 725–726.
Glasser, L. A. 1979 *Ph.D. thesis, Massachusetts Institute of Technology, Cambridge.*
Haus, H. A. 1975*a* *J. appl. Phys.* **46**, 2049–3058.
Haus, H. A. 1975*b* *IEEE J. Quantum Electron.* **QE-11**, 736–746.
Haus, H. A. 1975*c* *IEEE J. Quantum Electron.* **QE-11**, 323–330.
Haus, H. A. & Ho., P.-T. 1979 *IEEE J. Quantum Electron.* **QE-15**, 1258–1269.
Haus, H. A., Shank, C. V. & Ippen, E. P. 1975 *Optics Commun.* **15**, 29–31.
Ho, P.-T. 1978 Sc.D thesis, Massachusetts Institute of Technology, Cambridge.
Ho, P.-T., Glasser, L. A., Ippen, E. P. & Haus, H. A. 1978*a* In *Picosecond phenomena* (ed. C. V. Shank, E. P. Ippen & S. L. Shapiro). Berlin and Heidelberg: Springer-Verlag.
Ho, P.-T., Glasser, L. A., Haus, H. A. & Ippen, E. P. 1978*b* Presented at Topical Meeting on Picosecond Phenomena of the Optical Society of America, Hilton Head, South Carolina.
Ho, P.-T., Glasser, L. A., Ippen, E. P. & Haus, H. A. 1978*c* *Appl. Phys. Lett.* **33**, 241–242.
Ippen, E. P. & Shank C. V. 1977 *Topics in applied physics* (ed. S. L. Shapiro), ch. 3. Berlin, Heidelberg and New York: Springer-Verlag.
Kressel, H. & Butler, J. K. 1977 *Semiconductor lasers and heterojunction LEDs.* New York, San Francisco and London: Academic Press.
Kuizenga, D. J. & Siegman, A. E. 1970 *IEEE J. Quantum Electron.* **QE-6**, 694–708.
Kurokawa, K. 1973 *Proc. Inst. elect. Electron. Engrs.* **61**, 1386–1410.
New, G. H. C. 1974 *IEEE J. Quantum Electron.* **QE-10**, 115–124.
Paoli, T. L. 1975 *IEEE J. Quantum Electron.* **QE-11**, 276–283.
Shank, C. V. & Ippen, E. P. 1974 *Appl. Phys. Lett.* **24**, 373–375.

Discussion

T. S. Moss (*Royal Signals and Radar Establishment, Malvern, Worcs., U.K.*). Several papers have stressed the importance of 'noise-drive' in pulse formation and I therefore wonder if it would be interesting to add some fairly high-power, broad-band noise to the modulation input.

H. A. Haus. This is an interesting proposal. However, the experiment will be difficult if it is to yield quantitative information. Even though the spectrum of the source may be known, it is difficult to extricate the spectrum of the diode current, because of the parasitic impedances in the network containing the diode.

MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
THE ROYAL SOCIETY A
PHILOSOPHICAL TRANSACTIONS OF